



MATHEMATICS SPECIALIST : UNITS 1 & 2,

Tuesday 19th September 2023

Test 4 – (10%)

2.2.1-2.2.11, 1.1.2, 2.3.2-2.3.3

KB

Time Allowed
25 minutes

First Name

Surname

Marks

26 marks

Circle your Teacher's Name: Ms Chua Mr Koulianos Mr Liu Ms Robinson

Assessment Conditions: (N.B. Sufficient working out must be shown to gain full marks)

- ❖ Calculators: Not Allowed
- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

PART A – CALCULATOR FREE

1. [1, 2 and 1 marks]

Consider the matrices below:

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 4 & -1 & 5 \\ 1 & -2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -5 \\ -9 & 3 \\ 11 & 3 \end{bmatrix}$$

a. Why is BA not possible?

b. If $A^{-1} = \frac{-1}{16} \begin{bmatrix} 8 & -8 & 16 \\ -3 & -1 & y \\ x & 3 & -12 \end{bmatrix}$, give the values of x and y .

c. A third matrix C is such that $BC = \begin{bmatrix} 7 \\ 21 \\ -39 \end{bmatrix}$. Give the dimensions of C.

2. [2 marks]

Express $10.\overline{07}$ as an improper fraction.

3. [3 and 3 marks]

Janice claims that if two invertible matrices A and B are such that $AB = I$, where I is the Identity matrix, then A and B must be inverses of each other.

a. Is Janice's claim, correct? If so, explain why or if not explain why not.

b. Given that two matrices A and B are inverses of each other, simplify the expression below, showing each step.

$$A^{-1}B^2A^3B^{-1}$$

4. [2 and 3 marks]

a. Given Matrix $A = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}$, show that $A^2 = I$.

b. Given Matrix $B = \begin{bmatrix} x & 1+x \\ 1-x & -x \end{bmatrix}$, state any restriction on x , if any, for $B^2 = I$.

5. [4 marks]

Solve the equation to determine matrix B:

$$\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} B = B + \begin{bmatrix} 2 & 4 \\ 6 & 7 \end{bmatrix}$$

6. [3 and 2 marks]

- a. Determine the single 2×2 matrix which describes a rotation of 135° anticlockwise followed by another rotation of 30° anticlockwise.

- b. Hence find the values of $\sin\left(\frac{11\pi}{12}\right)$ and $\cos\left(\frac{11\pi}{12}\right)$

Reading Time: An initial 2 minutes to view BOTH sections



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Time Allowed 25 minutes	First Name	Surname	Marks 26 marks
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Circle your Teacher's Name: Ms Chua Mr Koulianos Mr Liu Ms Robinson

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- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

PART B – CALCULATOR ASSUMED

1. [3, 2 and 2 marks]

O(0,0), A(6,1) and B(5,3) are the vertices of a triangle.

- a. Triangle OAB is transformed to triangle OA'B', where A' is (6, 19) and B' is (5, 18), by transformation T_1 . Find and describe matrix T_1 .

- b. Triangle OA'B' is transformed by matrix $T_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to triangle OA''B''. Determine points A'' and B''.

- c. Find a single matrix that will transform triangle OA''B'' directly to triangle OAB.

2. [1, 1, 1, 2, 2 and 2 marks]

A brewer uses two different malts, M1 and M2, and a special ingredient, factor B, to produce two beers, Swan Bay and Crawley Bars. The number of units of each ingredient required to produce these beers are given in the table.

	M1	M2	B
1 litre of Swan Bay	4	3	1
1 litre of Crawley Bars	5	2	1

a. Write a matrix, A, which represents the information in the table.

Orders for Swan Bay and Crawley Bars are 20 000 litres and 30 000 litres respectively for the next production period.

b. Write a row matrix, B, representing the size of orders.

The cost of ingredients M1, M2 and B are 10 cents, 15 cents and 50 cents per unit respectively.

c. Write a column matrix, C, representing the cost of the three ingredients.

Using matrices, A, B and C and appropriate matrix operations, determine:

d. the matrix which represents the total number of units of M1, M2 and B required for the upcoming order.

e. the matrix which represents the cost to make 1 litre of each type of beer.

f. the matrix which represents the total cost of the upcoming order.

3. [4 marks]

Prove by contradiction that there are no positive integer solutions of x and y for the Diophantine equation $x^2 - y^2 = 1$.

4. [2, 2, 1 and 1 marks]

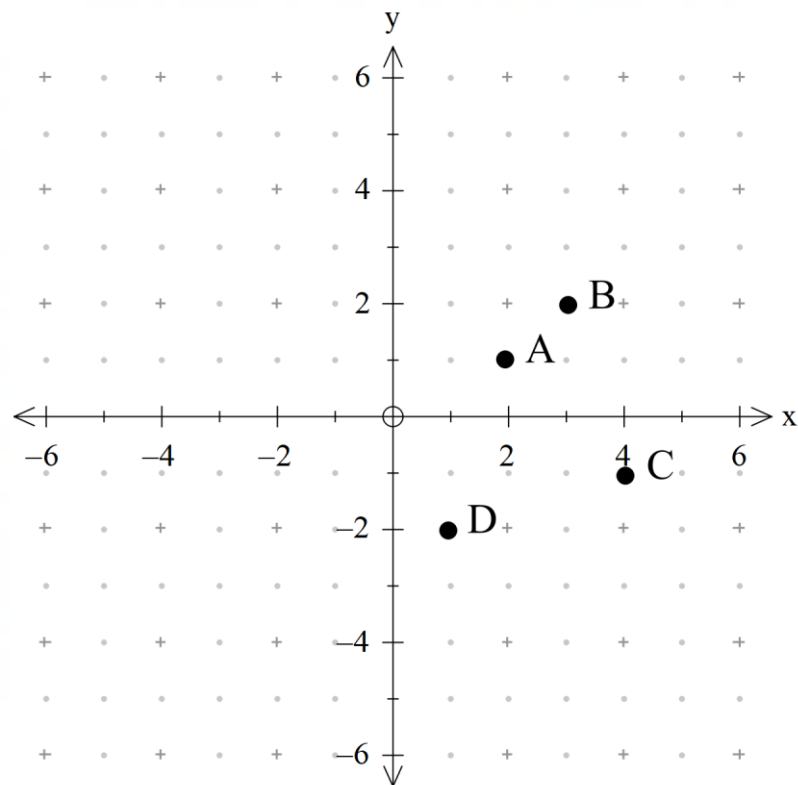
A quadrilateral ABCD has vertices:

A (2, 1)

B (3, 2)

C (4, -1)

D (1, -2)



a. Sketch on the axes given, the image of ABCD after the transformation S has been applied, where $S = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$.

b. Use words to describe the transformation(s)

c. If the area of ABCD is k units², state the area of its image under S.

d. State the matrix that would transform the image back to ABCD.